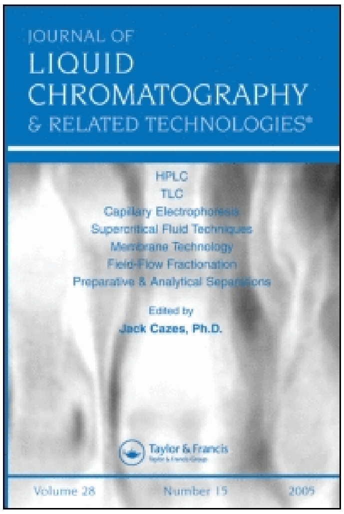

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THREE-DIMENSIONAL VELOCITY PROFILE IN CHANNEL WITH TRAPEZOIDAL CROSS-SECTION

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Field-flow fractionation is an analytical method based on simultaneous action of two gradients: a gradient of the solute concentration caused by an external field, and a velocity gradient of laminar flow of a carrier liquid caused by the viscosity effect. The two gradients are parallel to each other.

From the definition it follows that the carrier liquid is not only a transport medium but plays an active role in the separation process. A necessary condition for the theoretical description of the separation process is to know the velocity profile in the channel of a particular cross-section.

The velocity profile in the channel of rectangular cross-section was solved exactly by Cornish [1] and approximately (as a simple equation) by Takahashi and Gill [2]. Wičar [3] solved the velocity profile in the channel with triangular cross-section. Janča and Jahnová [4] made an attempt to describe the velocity profile in trapezoidal and parabolic cross-section channels.

Janča [5] recently presented one of the previous solutions [4] (see Eq.2) for three-dimensional demonstration of the velocity profile in trapezoidal cross-section channel (eq.6 and Fig.6b in ref.5). Unfortunately, the conclusions of Janča and Jahnová [4] concerning the velocity profile functions are incorrect. The functions do not satisfy the basic Navier-Stokes equation in spite of the fact that the authors presented this equation as a starting point of their considerations. This discrepancy was brought to attention by Wičar [3] already in 1988.

For a unidirectional, horizontal and steady-state laminar flow of an incompressible viscous liquid, the Navier-Stokes equations can be written as follows (the coordinate system is shown in Fig.1):

$$\nabla^2 \vec{u} = - \frac{G}{\mu} \qquad (1)$$

where ∇^2 is the Laplacean (in our case $\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$), $\vec{u}$ is the x-component of the velocity vector of the streamline, G is equal to $-\partial p/\partial x$ or approximately to $\Delta p/L$ (Δp is the difference between the inlet and outlet pressures), μ is the viscosity of the carrier liquid. The solution of Janča and Jahnová [4] for the trapezoidal cross-section channel is

$$u(y,z) = \frac{\Delta p}{2\mu L} b^2\left(1 - y^2/ b^2\right)\left(1 + \gamma z/c\right)^2 . f(z) \qquad (2)$$

where b is the channel half-width for z=0, c is the channel half-height for y=0, γ is defined as $(W_2-W_1)/(W_2+W_1)$, W_1 and W_2 are the widths of the channel bases, $f(z) = \left(1-\cosh(\sqrt{3}.a' z/c)/\cosh(\sqrt{3}.a')\right)$ where $a' = c/\left((W_2/2-b).z/c + b\right)$ (see Fig.1). The correction term $f(z)$, which was derived on the base of

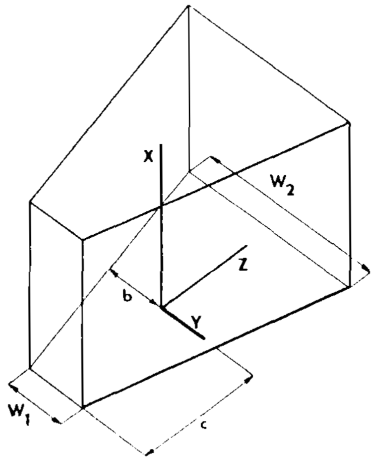

Figure 1
Scheme of the trapezoidal cross-section channel. There are drawn the coordinate system and the geometrical parameters of the channel used in text.

Takahashi and Gill's results [2], affects the final function only for $|z| \approx b$ and can be considered equal to 1 for our comparative purposes.

A simple derivation of Eq. 2 [without the term *f(z)*] accordingly to Eq. 1 yields the following result:

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = - \frac{\Delta p}{2\mu L} \left[(1+\gamma z/c)^2 - (b^2 - y^2)\gamma^2/c^2\right] \qquad (3)$$

that proves that Eq. 2 cannot be a solution of Eq. 1 because Eq. 3 is not equal to $-\Delta p/2\mu L$. The solution for the parabolic cross-section channel published in [4] exhibits similar errors. Moreover, Janča [5] has used Eq. 2 for calculation of resolution and consequently his conclusion has failed (Table 1 in ref. 5). In Fig. 2a one can see the course of Eq. 2 for functional values >0. It is obvious that the function reaches zero values

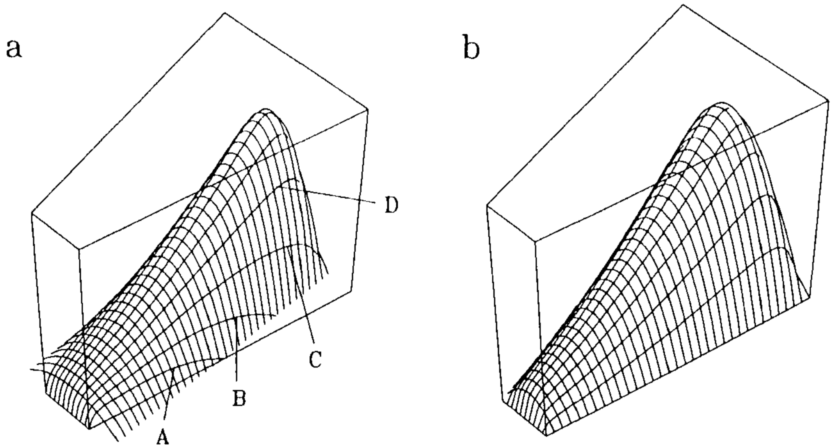

Figure 2
Comparison of the velocity profiles solved by Janča and Jahnová [4] (a - the course of Eq.2) and Pazourek and Chmelík [6] (b - the course of Eq.6). For demonstration, the Janča and Jahnová's function is plotted only for u(y,z)>0. Selected values: W_1=1, W_2=3, c=10 (it corresponds to the angle between the channel walls to 5.7°).

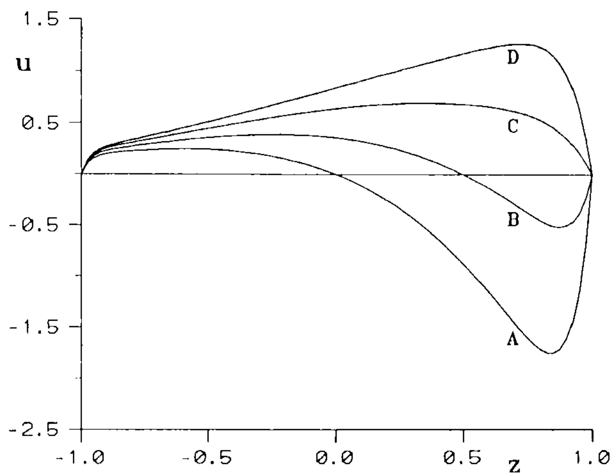

Figure 3
Detailed course of Eq.2. The values u of the vertical axis are relative velocities equal to u(y,z)/u(0,0). Positive parts (u>0) of the curves A,B,C, and D are identical to those drawn in Fig.2a as surface lines A,B,C and D of the same function.

outside the channel side walls at the narrower base of the channel cross-section and at the broader base it would have (not drawn) negative values (it means negative velocities) at the channel side walls.

Solutions that fulfill Eq.1 for channels of different cross-sections are derived and more widely discussed in [6]. For comparison, using the above mentioned notation the solution of the velocity profile for the channel of trapezoidal cross-section can be written as follows:

$$u(y,z)=\frac{\Delta p}{2\mu L}\, b^2 \frac{1}{1-\left(\frac{W_2}{2c}-\frac{b}{c}\right)^2}\left[\left(1+\left(\frac{W_2}{2b}-1\right)z/c\right)^2-y^2/b^2\right] \quad (4)$$

and with the Takahashi and Gill's correction we can get:

$$u(y,z) = \frac{\Delta p}{2\mu L}\, b^2 \frac{1}{1-\left(\frac{W_2}{2c}-\frac{b}{c}\right)^2} \cdot \left[\left(1+\left(\frac{W_2}{2b}-1\right)z/c\right)^2-y^2/b^2\right].f(z) \quad (5)$$

The course of Eq.5 is three-dimensionally drawn in Fig.2b for a channel of the same geometrical parameters as in Fig.2a.

The incorrectness of the Janča and Jahnová's solution [4] are evidently shown in Fig.3 where their function (Eq.2) reaches even negative values at the broader base of the channel cross-section ($z>0$ and $|y|>b$). Curves A,B,C,D are the same as in Fig.2a.

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